



New Approaches in 3D Geomechanical Earth Modeling

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Received: 10 Jan 2019; Accepted: 20 Jan 2019

DOI: 10.22107/JPG.2019.88410

Keywords

Asmari Reservoir, Geostatistical Estimation with Bayesian Inference, Markov Chain – Monte Carlo, Model Based Geostatistics, Ordered Weighted Averaging, Unconfined Compression Stress

Abstract

In this paper two new approaches for building 3D Geomechanical Earth Model (GEM) were introduced. The first method is a hybrid of geostatistical estimators, Bayesian inference, Markov chain and Monte Carlo, which is called Model Based Geostatistics (MBG). It has utilized to achieve more accurate geomechanical model and condition the model and parameters of variogram. The second approach is the integration of the models resulted of different estimators for more reliable-robust-accurate estimation, and using Ordered Weighted Averaging (OWA) data fusion. More accurate estimations help to achieve better results with less uncertainty in the stage of data fusion.

Ordinary Kriging (OK), Universal Kriging (UK), MBG and OWA were utilized for making 3D GEM of Unconfined Compression Stress (UCS) in a reservoir of an oil field in Dezful Embayment. The results were shown that the accuracy of MBG was twice of UK, whereas the model obtained of OK was unacceptable. The results of OWA were even 40% better than MBG.

1. Introduction

Different methods are developed for 3D modeling of geological, reservoir and geomechanical properties with different individual result. Consequently, depending on the number of modeling methods are used, different estimations in each block will achieve with different errors. Geostatistical (Almeida, 1999; Hu and Le Ravalec-Dupin, 2004), intelligent (Hamada and Elshafei, 2009; Cao et al., 2015; Karimi et al., 2016), fractal (Hewett, 1986; Hewett, 1993; Al-Zainaldin et al., 2017), and hybrid (Michael et al., 2010; Shiri et al., 2012) based methods are examples that are used to modeling different properties of a heterogeneous reservoirs. Other approaches like analytical, geometrical, statistical, conditional, and

numerical methods are also utilized for this purpose. Using chain estimators, especially in the case of lack of adequate data, increases the chance to access to a better model. Effective ranking of property modeling is the key in developing chain estimators (Tyagi et al., 2008 and 2009). Seismic attributes, petrophysical data, image logs, core and drilling data, in situ and lab mechanical tests are commonly data in developing a chain estimator. Scale incompatibility between various sources of the data is a challenge in using chain estimators, which are investigated in some researches (Masoudi et al., 2017; 2018). The geostatistical simulators are widely used in petroleum engineering. The reason is the capability of presenting different realizations, slightly avoid of smoothing geostatistical estimators,

achieving a more reliable static model, and considering fluid flow simulation (Deutsch, 2006; Pyrcz and Deutsch, 2014). Building 3D GEM using object based modeling methods, e.g. multiple point statistics, are also advised (Souche et al., 2015). Multiple point statistics, especially when seismic attributes are available, are going to be popular (Bavand Savadkoobi et al., 2018). However, their main disadvantage is significant uncertainty associated with training images (Mariethoz and Caers, 2014; Mariethoz, 2018). Some shortcomings were addressed in using geostatistical methods to make a 3D GEM (Deutsch, 2006; Rasouli and Tokhmechi, 2010). For instance, these methods have pointed to the limitations of variogram, and smoothing effect of Kriging, but they have offered alternative methods. Literature shows a significant increasing trend to build 3D GEM by using intelligent methods. Multi-Layer-Perceptron (MLP), Radial Basis Function (RBF), Wavenet, and Wavelet Neural Network (WNN) are examples of the networks were recently utilized to estimate 3D GEM (Zhang and Bevenista, 1992; Fang and Chow, 2006; Alexandridis and Zapanis, 2013; Tokhmechi et al., 2018a; 2018b).

Because of the problems associated in making 3D GEM by different methods, comparison between the applications of the methods have been also investigated (Abdideh and Ghasemi, 2014). Abdideh and Ghasemi have not claimed that the results of research is generalized. Kamali and Ostad et al., have investigated the challenges of building 3D GEMs in carbonate reservoirs and have reported valuable approaches to make more reliable models (Kamali et al., 2013; Ostad et al., 2018).

Model based geostatistics that has rarely used in geosciences, is an estimator, which conditions the results to the geological and tectonical behavior of the field. Literature confirms that MBG has been effective estimator in other disciplines (Diggle and Ribeiro, 2007). In the current paper, two different approaches were used to estimate mechanical properties. Applicability of MBG to making a 3D GEM and sensor data fusion, which is a powerful approach to integrate different models, were used to achieve more robust GEM. Previous studies have shown that, the results of developing 1D GEM using data fusion have been wonderful (Tokhmechi et al., 2009; Soroush et al., 2010). The applicability of data fusion to build 3D GEM is also investigated in current paper.

2. Methodologies

In this section, model based geostatistics and ordered weighted averaging are briefly introduced.

2-1 Model Based Geostatistics

Suppose that \underline{X} is the input matrix and \underline{T} the output vector based on the equations (1) and (2):

$$\underline{X} = \begin{bmatrix} x_1^1 & \cdots & x_1^m \\ \vdots & \ddots & \vdots \\ x_n^1 & \cdots & x_n^m \end{bmatrix} \quad (1)$$

$$\underline{T} = \begin{bmatrix} t_1 \\ \cdot \\ \cdot \\ t_n \end{bmatrix} \quad (2)$$

where x_i^j is the input j in the block i . Block

coordinates (x , y and z) and any continuous properties could be the input data, similar to seismic attributes. n and m are the number of blocks and continuous properties respectively, that might be estimated. t_1 to t_n are observed outputs.

For example in schematic reservoir, five wells were drilled (Figure 1). Assume that 1D GEMs in all wells were developed and UCSs with 15 cm resolution were estimated. Suppose that the final goal is estimation of UCS in reservoir's lattice. In this case, t_n stands for estimated UCSs in the wells.

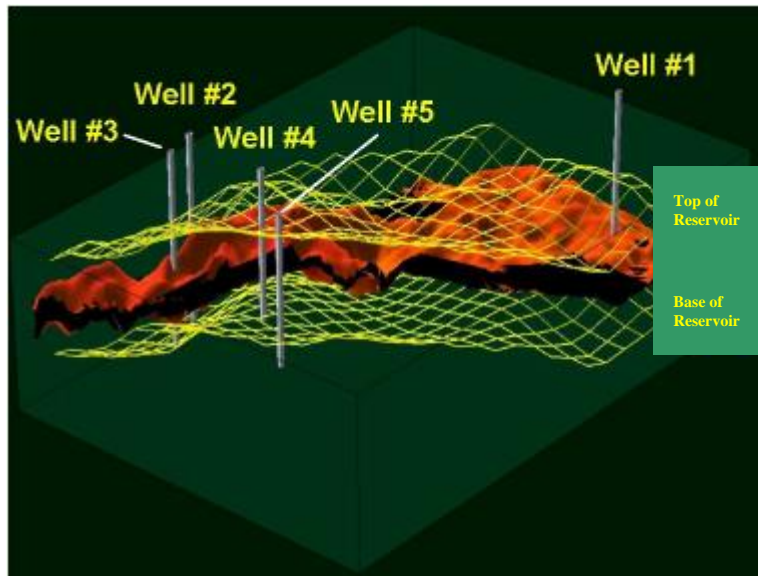


Figure 1. Schematic location of five wells, which have been drilled on a reservoir, and designed lattice for making 3D GEM.

In Figure 1, the coordinates of each data in the wells are considered as x_i^1 , x_i^2 and x_i^3 . Suppose two seismic attributes as well as porosity are available as continuous input data. Assume that seismic attributes have been recorded, and porosity have been estimated in each block. Consequently, input matrix also consists of x_i^4 , x_i^5 and x_i^6 . Also assume that a linear trend was fitted to the UCS data:

$$\begin{aligned} T_i &= UCS_i \\ &= \beta_0 + \beta_1 x_i^1 + \beta_2 x_i^2 \\ &+ \dots + \beta_m x_i^m + \varepsilon_i \end{aligned} \quad (3)$$

where, in hypothetical problem, m is equal to 6, and x_i^1 to x_i^6 are x , y , z , 1st and 2nd

seismic attributes and porosity respectively. x_i^1 to x_i^6 , must be available in all blocks. β_0 stands for intercept and β_1 to β_m (β_6), slopes or weights of the input data. ε_i is random residual, which might be estimated in each block.

The first step is to model the data trend based on the observations. Weights in equation 3, must be optimized in this step. The average of the residuals ε is usually equal to zero. Quasi-stationary hypothesis is infeasible and their estimation using geostatistical methods could be possible.

It will be possible to fit a trend surface to all blocks, represented in Figure 1. It should

be attention that x_i and β have to be determined in this step; so, fitting the trend surface must be possible. Residuals, in each block, might be estimated using geostatistical methods. Usually ordinary Kriging (OK) is suitable for this purpose.

Variogram modeling and optimization of its parameters (range (a), nugget (C_0) and sill (C)) are the most important parameters for estimation of residuals. θ and β are the parameters of variogram and trend, respectively. They have to be optimized in this step. Assuming that aforementioned tasks have done, UCS is so far estimated using universal Kriging (UK).

When limited database is available, variogram modeling and optimization of parameters are problem and subjective. Based on the models, which have been applied, and the person who modeled data there would be different results. MBG is an approach helps to achieve a more reliable/robust model. In literature of MBG, UK is called likelihood (Diggle and Ribeiro, 2007) according to the equation (4):

$$L(\zeta|T) \quad (4)$$

where T is observed feature space (outputs in equation 2). For instance, in this paper T is equivalent with UCS resulted from 1D GEM. ζ is represents the parameters might be optimized, and is defined by the equation (5):

$$\zeta = (\beta, \theta) \quad (5)$$

As a result, likelihood is in fact estimation using UK, condition to the observed data. In reality presenting an approximation of ζ is possible:

$$\hat{\zeta} = (\hat{\beta}, \hat{\theta}) \quad (6)$$

where $\hat{\zeta}$, $\hat{\beta}$ and $\hat{\theta}$ are approximation of the nominated parameters. Final estimation or posteriori in Kriging with Bayes inference (MBG), achieves from multiplication of likelihood in priori (Yamamoto, 2007; Brown, 2015; Diggle and Giorgi, 2015):

$$\Gamma(\zeta|T) \propto L(\zeta|T) \pi(\zeta) \quad (7)$$

In which $\pi(\zeta)$ and $\Gamma(\zeta|T)$ are priori and posteriori respectively. As aforementioned, estimation using UK was called likelihood. Estimation variance, which in UK just depends on residuals (ε), which can also be calculated. Therefore, it will be possible to give a Probability Density Function (pdf) for likelihood of each block (Figure 2).

Priori is also the results of UK. However, in the case of priori, the parameters of the variogram (range, nugget, and sill) must achieved from similar reservoirs. Suppose that 3D GEM for UCS has been built in the similar reservoirs, where their database is more completed than the currently studying reservoir. The average of variogram parameters of similar reservoirs, which must be extracted and utilized for making a new 3D GEM, is called priori. In the case of UCS at Dezful Embayment, the directions of stresses are well-known (Figure 3). Considering these information in modeling of UCS is the priori in fact.

A schematic pdf of the priori for an example block is presented in Figure 2. Posteriori ($\Gamma(\zeta|T)$) or in fact MBG, achieves from multiplying of likelihood ($L(\zeta|T)$) in priori ($\pi(\zeta)$) (Figure 2).

The remained question is how should multiply the priori in likelihood? In fact, this multiplication is a complex and time consuming procedure. For example equation of likelihood is as follows (Diggle and Ribeiro, 2007):

$$L(\beta, \theta|T) = f(T|\beta, \theta) = \frac{1}{(2\pi)^{m/2} |\underline{\underline{\Sigma}}(\theta)|^{1/2}} \exp\left(-\frac{1}{2}(\underline{T} - \underline{X}^t \underline{\beta})^t \underline{\underline{\Sigma}}(\theta)^{-1} (\underline{T} - \underline{X}^t \underline{\beta})\right) \quad (8)$$

where $\underline{\underline{\Sigma}}(\theta)$ is the covariance matrix of variogram, or in another words, the variogram matrix of observed samples for residuals in estimation of each block. The

approximation for trend is $\underline{X}^t \underline{\beta}$. \underline{T} represents the observed UCS. The number of parameters utilized for trend modeling shows by m .

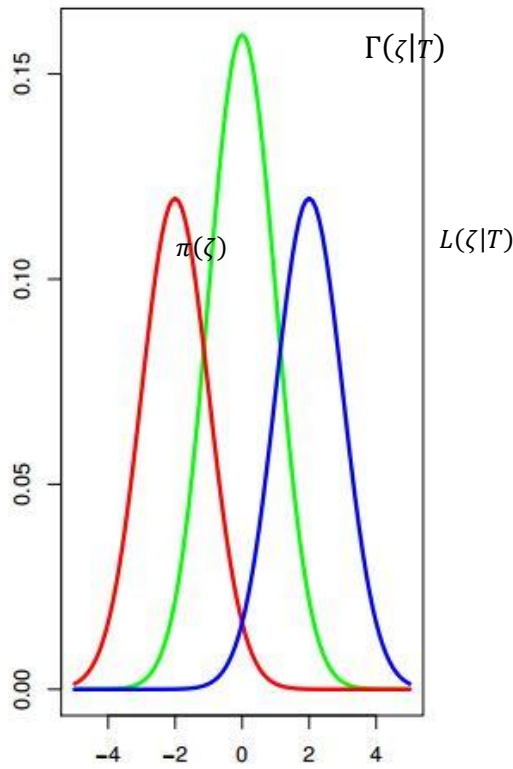


Figure 2. Schematic image of multiplication of pdfs of likelihood in priori, and resulted posteriori (MBG).

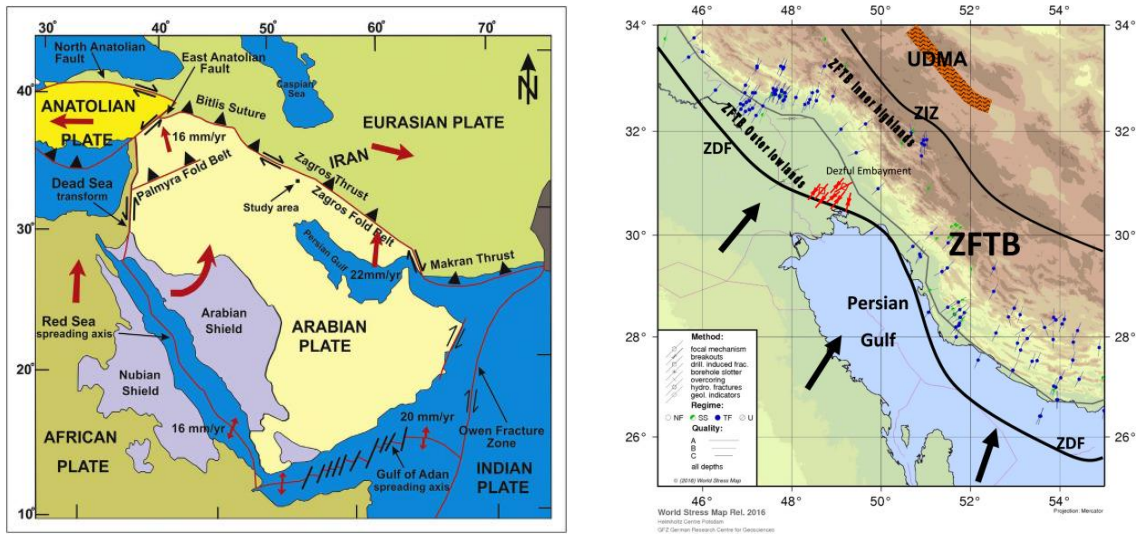


Figure 3. Stress direction in Dezful Embayment, might be utilized for approximation of the priori.

An equation similar to equation 8 has to be developed for priori, while its $\underline{\underline{\Sigma}}(\theta)$ differs with likelihood. Both equations must be found for each block, then multiplied to each other, which is complex and time consuming process. Approximation of the posteriori using Markov Chain – Monte Carlo (MCMC¹) is the well-known presented solution (Diggle and Ribeiro, 2007).

$$w_j \in [0,1] \tag{9}$$

$$\sum_{j=1}^e w_j = 1 \tag{10}$$

$$F(a_{k1}, a_{k2}, \dots, a_{ke}) = \sum_{j=1}^e b_{kj} w_j = b_{k1} w_1 + b_{k2} w_2 + \dots + b_{ke} w_e \quad k = (1, n) \tag{11}$$

2.2 Ordered Weighted Averaging

Ordered weighted averaging (OWA) is an effective fuzzy fusion method introduced by Yager (1988). An OWA operator of dimension n (the number of estimates using different estimators) is a mapping $F : R^n \rightarrow R$, that has an associated n vector, $w_i = (w_1 ; w_2 ; w_3 ; w_4 ; \dots ; w_k)$. For each i , where $1 \leq i \leq n$, the following equations are correct:

¹ MCMC is well described in: Diggle and

Ribeiro, 2007. Enthusiasts should review to the reference for details.

where b_j is the j^{th} largest element of the bag (a_1, \dots, a_n) , and k represents various blocks in the reservoir.

$$Min_i[a_i] \leq F_w(a_1, a_2, \dots, a_n) \leq Max_i[a_i] \quad (12)$$

Output of OWA, is always in the range of fuzzy operators *OR-AND* (Hsu and Chen, 1996; Kuncheva and Krishnapuram, 1996).

$$orness(w) = \frac{1}{n-1} \sum_{i=1}^n (n-i)w_i \quad (13)$$

In order to minimize error (e), w_i has to be optimized:

$$SSE = \left(\sum_{k=1}^e (b_{k1}w_1 + b_{k2}w_2 + \dots + b_{ke}w_e - d_k)^2 \right) \quad (14)$$

where dk represents real UCS in depth k of the wells, which their 1D GEM is available. In Optimistic OWA, w_i is defined as a function of Orness coefficient (α). α characterizes the degree in which the aggregation is like *or* (Max) operation (Yager, 1988):

$$\begin{aligned} w_1 &= \alpha; & w_2 &= \alpha(1-\alpha); & w_3 &= \alpha(1-\alpha)^2; & \dots; \\ w_{e-1} &= \alpha(1-\alpha)^{e-2}; & w_e &= (1-\alpha)^{e-1}; & & & 0 \leq \alpha \leq 1 \end{aligned} \quad (15)$$

In pessimistic OWA, w_i is defined as another function of Orness coefficient (α):

$$\begin{aligned} w_1 &= \alpha^{e-1}; & w_2 &= (1-\alpha)\alpha^{e-2}; & w_3 &= (1-\alpha)\alpha^{e-3}; & \dots; \\ w_{n-1} &= (1-\alpha)\alpha; & w_n &= (1-\alpha); & & & 0 \leq \alpha \leq 1 \end{aligned} \quad (16)$$

To find the optimum e , α should vary from 0 to 1 and e should be recalculated. The α which minimizes e , shows optimum w_i .

3. Case Study of the Application of the Methods

Ordinary Kriging (OK), Universal Kriging (UK), Model Based Geostatistics (MBG) and Ordered Weighted Averaging (OWA) with Optimistic (OOWA) and Pessimistic (POWA) mechanisms were applied for making a 3D GEM of UCS in one of the carbonate reservoirs in southwest of Iran. In this section the results of the models and comparison are presented.

3.1 Model Based Geostatistics

In the current study, modeling of priority was impossible because the models of similar reservoirs was not available. Inevitably, a portion of the database was considered for modeling of priority, and the rest for likelihood. Available well-data are represented in Figure 4. The UCS data for seven wells were available. The data of two wells (black circles) were selected to modeling the likelihood, and the rest (red circles) for priori.

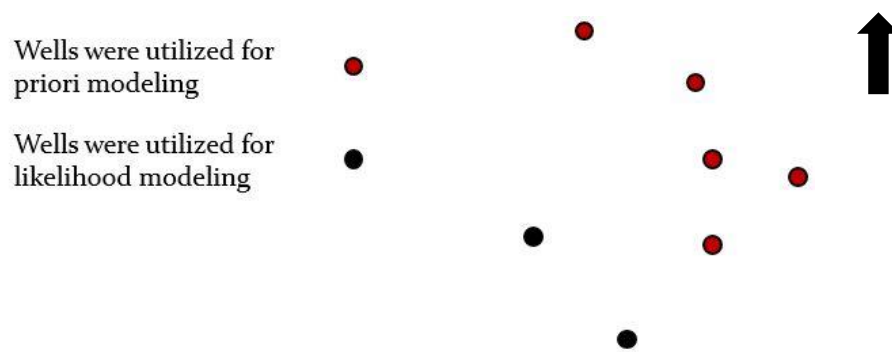


Figure 4. Schematic image of multiplication of pdfs of likelihood in priori, and resulted posteriori (MBG).

In geomechanical data, the highly possibility of existence of the trend must be encountered in geostatistical modeling. The reason is that majority of geomechanical properties differ with changing of lithology, depth, or even horizontal directions. Therefore, investigation of the existence of the trend, even before studying the distribution of

the data, is advised. In the case of existence of trend, the distribution of residuals might be investigated. The variability of UCS in one of the studied wells is plotted in Figure 5.a. Based on this log, UCS increases with depth. The probability distribution function (PDF) of the UCS (Figure 5.b) shows their bimodal distribution.

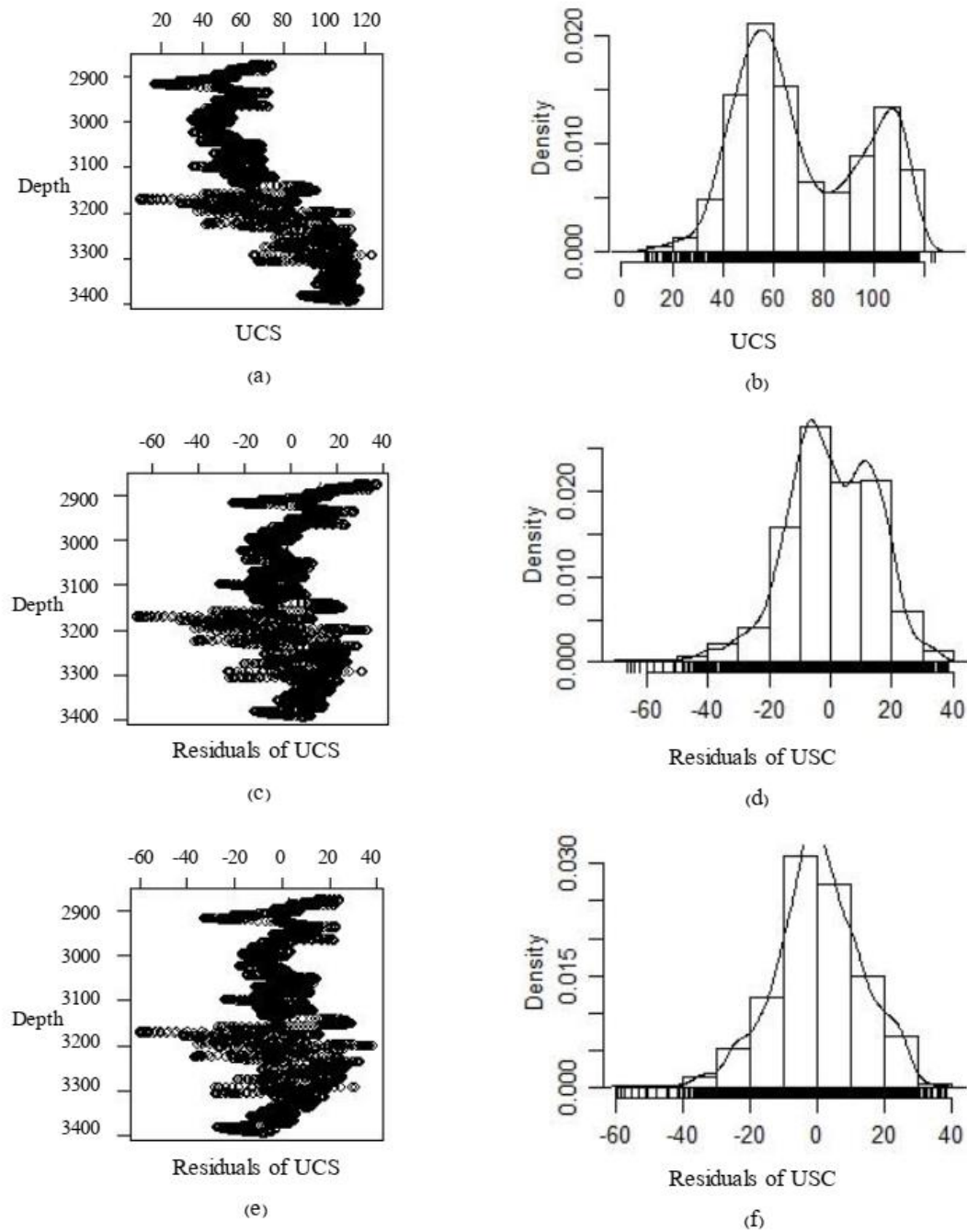


Figure 5. UCS log in Asmari reservoir of in one of the wells located in Dezful embayment (a) and associated PDF (b). The residuals of UCS after applying linear trend equation (c) and the PDF of residuals (d). The residuals UCS after applying quadratic trend equation (e) and their PDF (f).

Residuals after subtracting linear trend from UCS data (Figure 5.c) shows that trend somewhat removed. PDF of these data (Figure 5.d) shows that the distribution of residuals has approached to normal distribution, while bimodal behavior is remained.

Above procedure has replied with fitting quadratic trend function. Residuals and their PDF are plotted in Figures 5.e and 5.f respectively. The results shown that trend was removed, and PDF of residuals is almost Gaussian. As a result, UCS must be modeled using UK. For this purpose, quadratic trend were fitted and OK applied on the residuals.

The calculated variograms in depth direction, and fitted model are displayed In Figure 6. Variogram shows a continuity in this direction.

In Figure 7 the wireframe that UCS is modeled is displayed. The size of the blocks was considered equal to 50 - 50 m² (Figure 7.a). As the blocks outside the reservoir have to be removed, the border of the reservoir is

plotted in Figure 7.b.

In this study 3D GEM of UCS were modeled by using the following three methods:

- **OK**: without considering the trend, and after applying a non-linear transformation over the data to achieve a semi-Gaussian distribution. Inverse transformation were applied on the estimated results.
- **UK**: with considering a quadratic trend equation, and modeling the residuals using OK. In this case, PDF of the residuals were normal (Figure 5.c); therefore, no transformation was applied on-the data.
- **MBG**: the basic method was UK, and priori also was modeled using UK.

30 percent of UCSs of both studied wells were randomly selected as test data. These data were estimated using three methods, and the cross correlation between estimated amounts with real UCS were plotted in Figure 8.

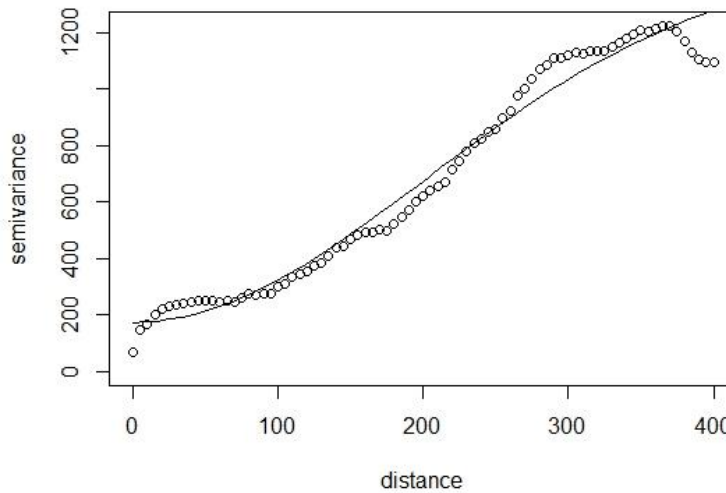
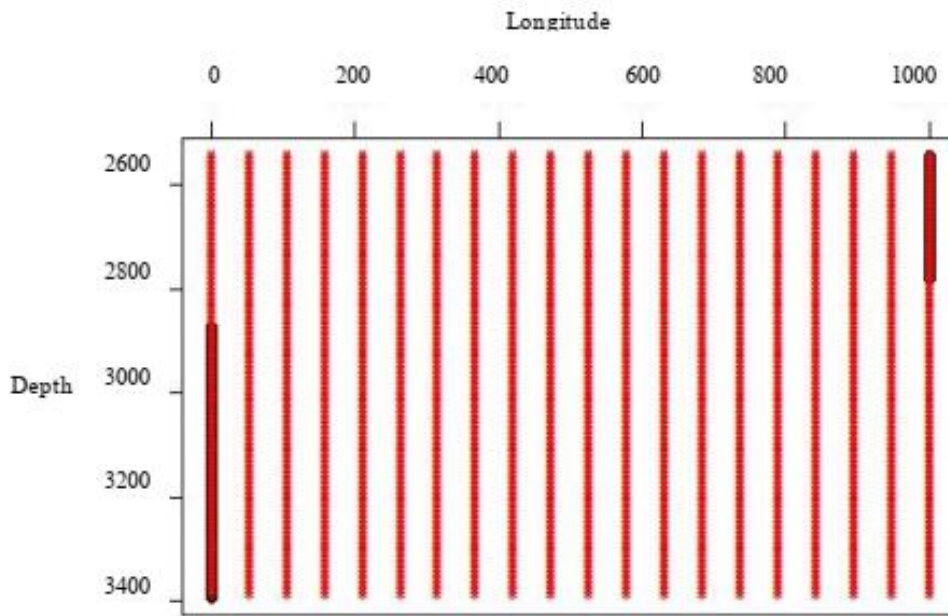
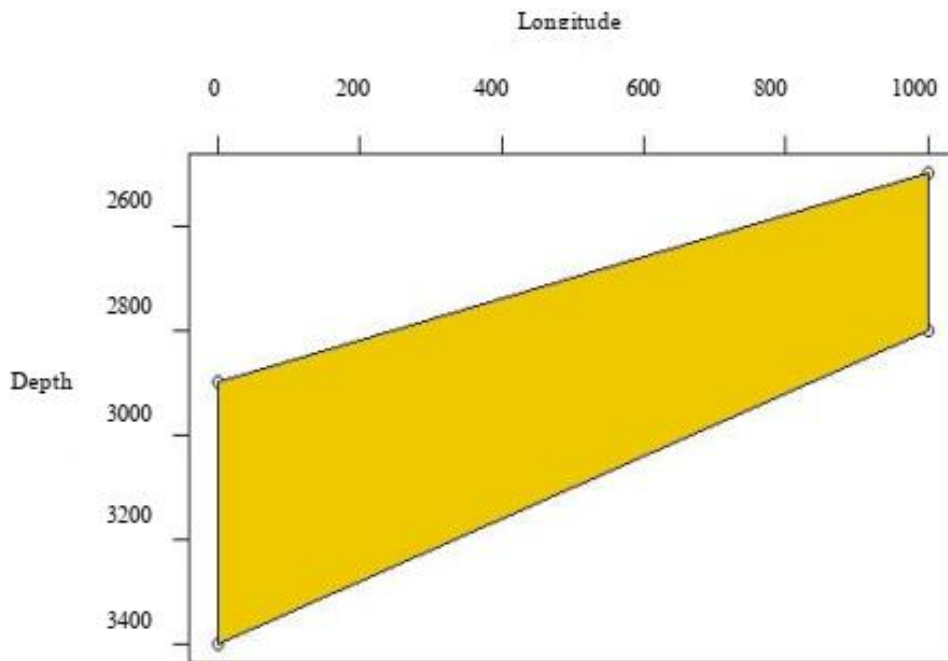


Figure 6. Vertical variogram of UCS and Gaussian fitted model. Range, nugget and sill are estimated equal to 300 m, 170 MPa², and 1115 MPa² respectively.

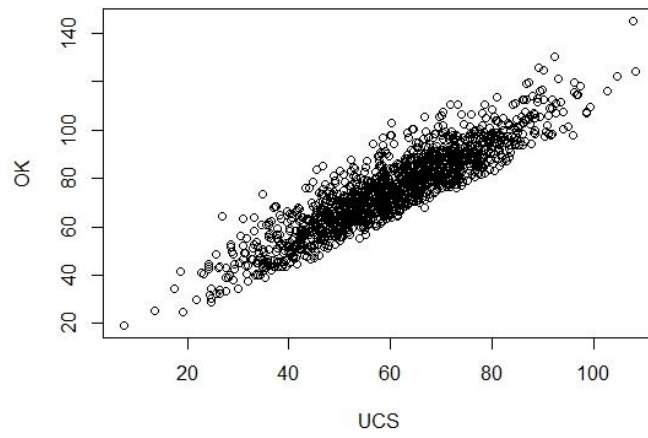


(a)

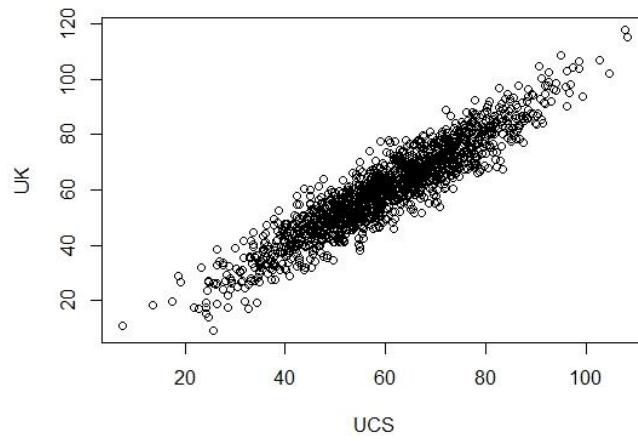


(b)

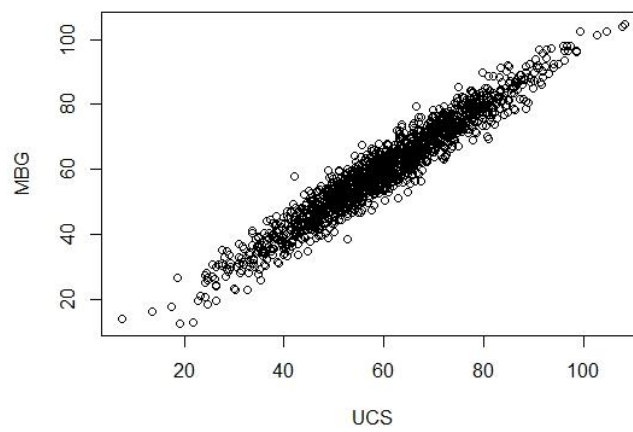
Figure 7. a) A section of the reservoir and designed lattice. The location of the wells and border of the reservoir is displayed (b).



(a)



(b)



(c)

Figure 8. Cross correlation of estimation of UCS using the methods a) OK, b) UK and c) MBG.

Based on Figure 8, there are significant difference between estimations using three methods, as follows:

- The cloud of error around line UCS=OK (45 degree) is wide (Figure 8.a (OK)), which means relatively high error. Also, the line trend of cross correlation oblique to higher amounts, shown that results are biased. Bimodal distribution of the data (Figure 5.b) might be the reason of bias estimation.
- Not only the cloud of error is narrower (Figure 8.b), but also the precision of the results is better and biasness is lower than OK. It is

obvious that the results of estimation using UK is better than OK.

- In the case of MBG, the biasness was almost be removed, and error cloud is obviously narrow, so its application is visually better than MBG.

It might be mentioned that high CPU processing time is the main disadvantage of the MBG. For instance, in Table 1, CPU processing time of three methods were reported. Correlation coefficient between estimated and real data, sum of error (SE), corresponds with precision or biasness, and sum of squared error (SSE) which shown the accuracy, were also reported in Table 1.

Table 1. Comparison between implementation of three estimators were used for modeling of UCS, from the aspect of CPU processing time, correlation coefficient, sum of error and sum of squared error.

Method	CPU Processing Time (S)	Correlation Coefficient	Sum of Error (MPa)	Sum of Squared Error (MPa ²)
OK	112	0.905	22384	408472
UK	98	0.929	310	53558
MBG	447	0.967	-22	23748

Based on Table 1, results of OK are obviously unacceptable. Also comparison between UK and MBG (Table 1) shown that MBG's estimations were more reliable than UK. However, MBG's CPU processing time is high. This will create highly problem when a giant reservoir must be modeled. INLA is the extension on MBG to decrease the time, which is not described in current paper (Zhang et al., 2016; Krainski et al., 2018).

3.2 Ordered Weighted Averaging

As previously described, integration of the results of different estimators to achieve more robust/reliable model, is the aim of applying of OWA. Therefore, OWA was applied over their results. In this section, the results of application of OWA over test data was presented. It should be mentioned that 30 percent of the data were randomly selected for this purpose. For simplicity sake, all test

data were considered as continues log. In Figure 9, estimated versus real UCS were plotted.

Real UCS considered as desired result, and the results of OK, UK and MBG were fused using OWA. In Figure 10 the results of optimization of α using optimistic (OOWA) and pessimistic (POWA) mechanisms were displayed. Processing of OWA operator is usually so fast. For example in current study, processing time was less that one second. In Table 2, correlation coefficient between real and estimated UCSs, as well as SE and SSE of estimated UCSs, for MBG, OOWA and POWA were reported. In the case of using OWA, in comparison with MBG, SSE has fallen down about 40 %. (Table 2). The estimation was unbiased with acceptable precision. Cross validation between real UCS and estimated ones using OWA (Figure 11) were shown highly acceptable results with narrow error cloud and slope of correlation

line around 45 degree.

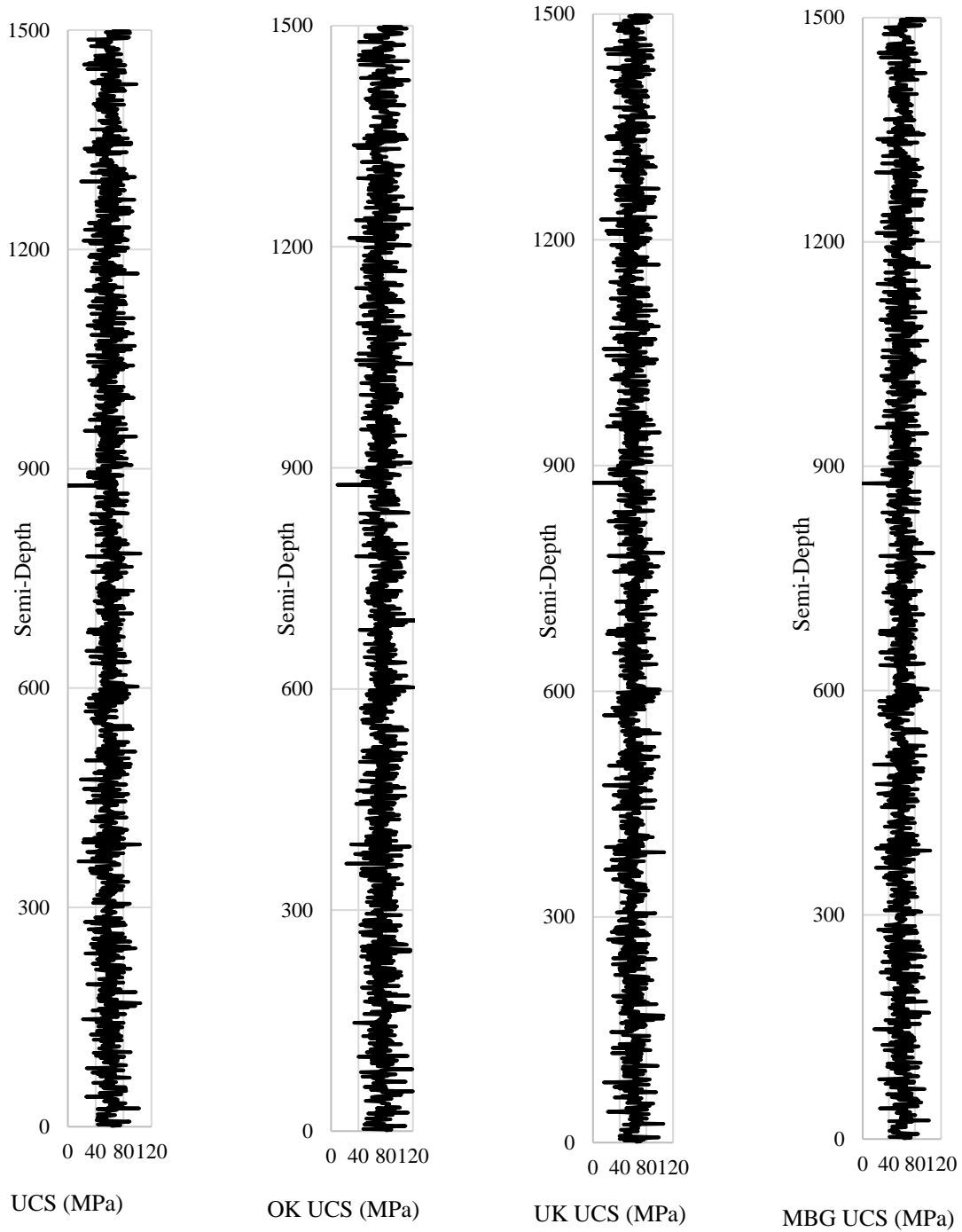


Figure 9. Real versus estimated UCS using three methods: OK, UK and MBG for the test data.

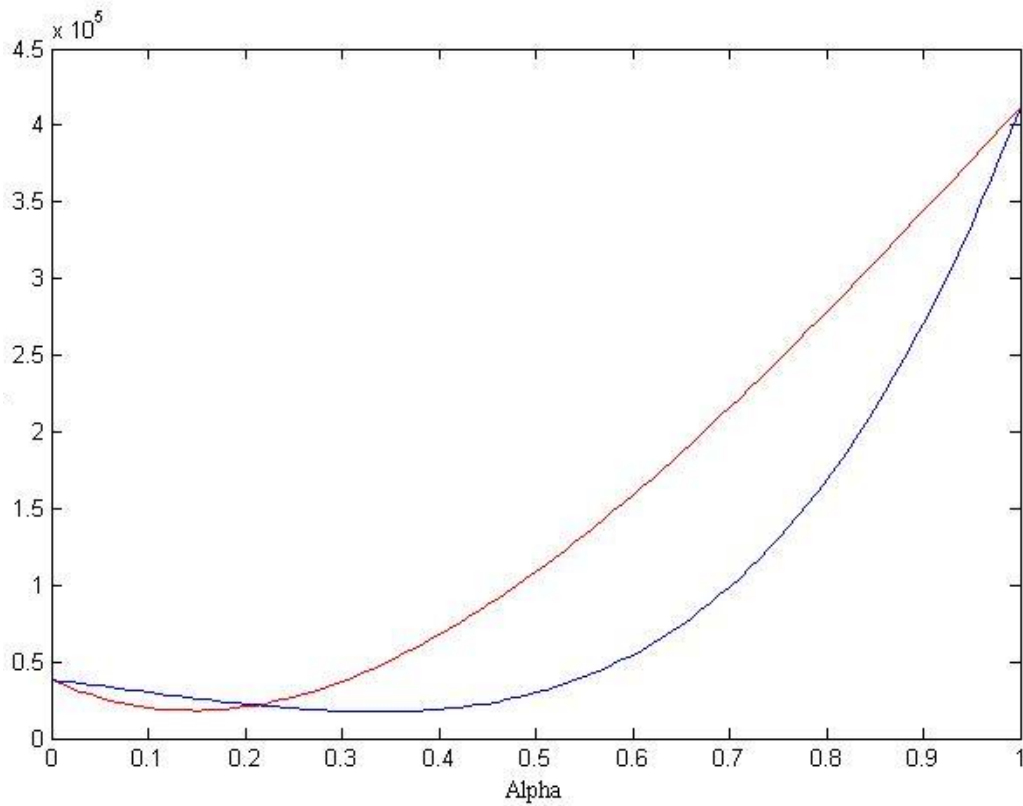


Figure 10. The results of optimization of the weights of OWA using Yager mechanisms.

Table 2. Comparison of the application of MBG with optimistic and pessimistic OWA in modeling of UCS, from the aspect of correlation coefficient, sum of error and sum of squared error.

Method	Correlation Coefficient	Sum of Error (MPa)	Sum of Squared Error (MPa ²)
MBG	0.967	-22	23748
Optimistic OWA	0.952	-24	18213
Pessimistic OWA	0.964	-19	17249

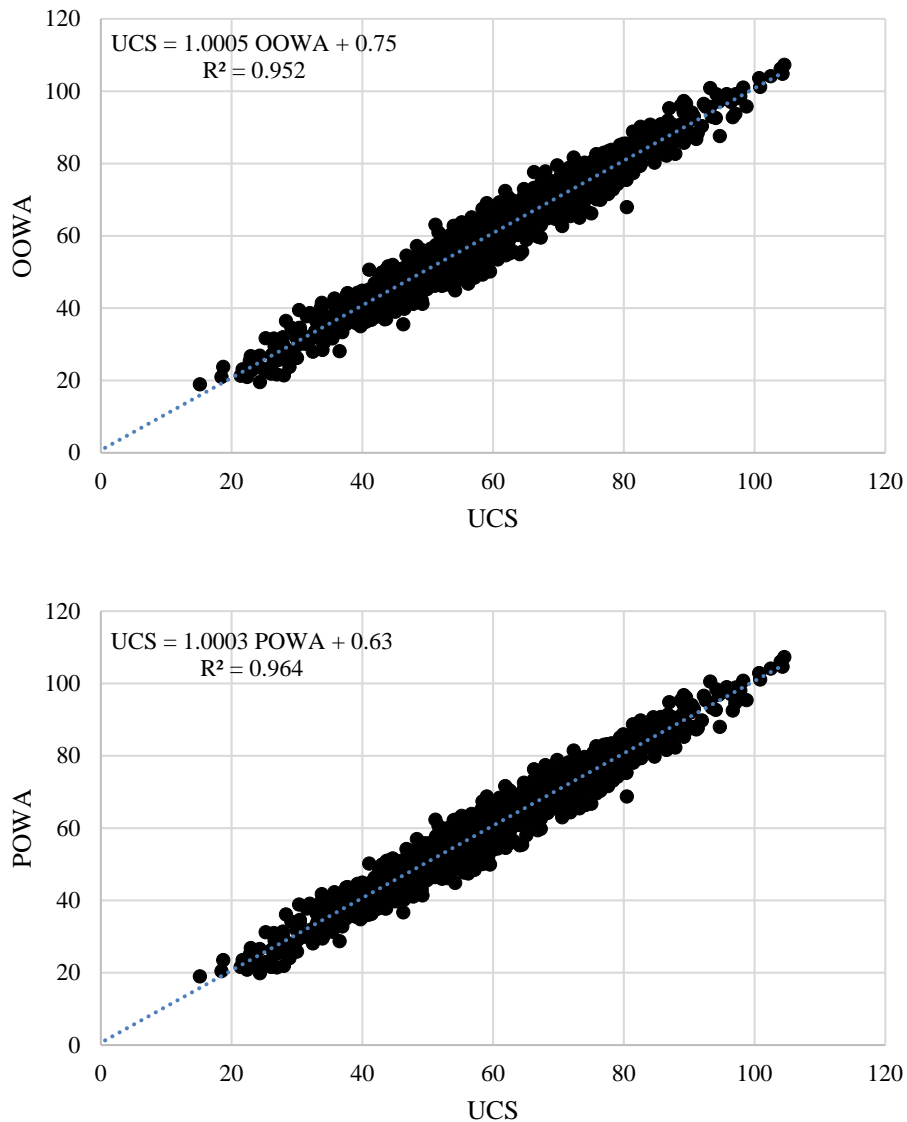


Figure 11. Cross validation of estimation of UCS using OWA with mechanisms a) optimistic and b) pessimistic.

To better show how OWA works, some parts of calculations for pessimistic mechanism were shown (Figure 12). Based on the Figure 10 optimum α in pessimistic mechanism is equal to 0.35. Therefore, optimum weights (three weights) must be calculated using equation 16. It should be repeated that the results achieved from three estimators (OK,

UK and MBG) might be fused together.

Weights are calculated as follows:

$$w_1 = \alpha^{n-1} = 0.35^{3-1} = 0.1225$$

$$w_2 = w_{n-1} = (1 - \alpha)\alpha = 0.65 \times 0.35 = 0.2275$$

$$w_3 = w_n = (1 - \alpha) = 0.65$$

In Figure 12, the highest estimation was highlighted with light gray, the middle with light brown and the rest without shading. Then results were descending sorted. Sorted results were inner product into optimized weights. Summation of the outcomes is called POWA. One by one comparison between POWA and also MBG with real UCS, shown that results of POWA were more robust and compatible with real UCS.

4. Conclusion

Building a 3D geomechanical model is a complex procedure, and existence of trend for mechanical properties is anticipated, also their behavior is usually highly heterogeneous. In the current paper, two new methods, MBG and OWA were introduced and could help to achieve more reliable 3D

geomechanical model. Application of the methods to modeling UCS over two wells in a reservoir located in south-west of Iran were investigated. Comparison between the results with OK and UK shown that the accuracy and precision of newly introduced methods were considerably better than common estimators. By the meanwhile, it observed that running of MBG is time consuming; therefore, it probably will not be feasible to build a 3D geomechanical earth model using MBG. INLA, is a recently developed extension of MBG, which runs so faster. Researchers are advised to focus on INLA as a powerful estimator to make 3D GEM in giant reservoirs.

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