

Original article

A New Thermo-Hydro-Mechanical Model in Unsaturated Deformable Soils

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Abstract

Exposure of unsaturated clay soils to elevated temperatures can significantly alter their volumetric behavior, hydraulic properties, and shear strength. The need to understand the thermal response of soils arises in a wide range of geoenery engineering applications, including deep geological disposal of nuclear waste, geothermal structures, oil and gas drilling, and thermal ground improvement techniques. In particular, for underground nuclear waste repositories, a comprehensive understanding of the water-retention characteristics, hydro-mechanical interactions, and temperature-dependent behavior of the surrounding soil is essential for preventing the migration of radioactive contaminants. Although numerous studies have investigated the influence of temperature on soil behavior, the response of unsaturated soils to thermal changes remains highly complex due to the strong coupling among thermal, hydraulic, and mechanical processes. Therefore, a more comprehensive understanding of these coupled mechanisms is required. In this study, a thermo-hydro-mechanical constitutive model is developed to predict the behavior of unsaturated clay soils under varying temperature conditions. The proposed model is based on the constitutive framework developed by the first author and is extended to incorporate temperature-dependent effects. The model is capable of capturing the coupled influence of temperature on both the stress-strain response and hydraulic behavior of unsaturated soils. The plasticity based on subloading and bounding surfaces is reformulated by incorporating temperature as an additional state variable, enabling the simulation of thermo-mechanical and thermo-hydraulic responses, respectively. The coupled thermo-hydro-mechanical behavior is subsequently described through appropriate hardening laws. The proposed formulation accounts for temperature-induced changes in the hydraulic boundary curves and enables the investigation of thermal plastic strains and their influence on the mechanical response of both normally consolidated and overconsolidated soils. In addition, the model considers the coupled effects of temperature and matric suction, thereby providing a framework for simulating the thermo-hydraulic response of unsaturated soils. Finally, the predictive capability of the proposed model is evaluated against experimental results obtained from modified triaxial tests conducted on unsaturated soils under different thermal conditions. The comparison demonstrates the ability of the proposed formulation to reproduce the main features of the observed thermo-hydro-mechanical behavior and confirms its potential for analyzing temperature-dependent responses of unsaturated soils in engineering applications.

1. Introduction

Temperature changes affect soil behavior fundamentally and this effect is due to soil deformations at high temperatures. In other

words, changes in soil structure and changes in soil volume occur under the effect of thermal loading. When thermal volumetric contractions in the soil do not fully reversible after cooling, thermal contractions are called irreversible and

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are sometimes called thermal collapse phenomena. In addition to the necessity of investigating the hydromechanical behavior of the soil [1,2], due to the importance of soil behavior prediction, many experimental and numerical research has been done on soil behavior that can show reversible or irreversible deformations under non-isothermal conditions (e.g., [3, 4]). But the fact is that the behavior and reaction of soil to temperature changes are complex and difficult and require a better and more accurate understanding of the mechanism and factors affecting this field.

The soil skeleton is composed of solid part and air pressure in addition to the bulk water and meniscus water. These two parts cause different behavior of air pressure and water pressure and consequently cause a change in the mechanical behavior of the soil. So in addition to suction, the degree of saturation also affects the stress-strain behavior in unsaturated soils. Therefore hypothetical model must consider the combined effects of hydraulic hysteresis behavior and mechanical behavior, in which plastic changes in saturation degree affect the stress-strain behavior and subsequent plastic volume strains on the water characteristic curve of unsaturated soils. Investigation of soil hysteresis behavior is require for predicting and interpreting the hydro mechanical behavior of unsaturated soils in the wetting and drying process. Therefore, models that are able to describe such soil behavior are effective in the development of unsaturated soil models .

Temperature change has a considerable effect on liquid-gas interfacial tension, which by increasing temperature it linearly decreases [5]. For describing the hydraulic behavior under different temperature and further the hydro-mechanical coupling, the dependency of hysteresis behavior on temperature should be modelled. In the past decades, several models have been adopted to demonstrate temperature effect on SWRC and other hydraulic parameters [3, 6-11]. In addition to the above, the effects of temperature change on soil deformation have been investigated [4, 12-16]. Since in saturated soils, the change in volume due to temperature changes is independent of stress, in unsaturated soils by assuming suction as internal stress, it can be said that temperature cause volume changes independently of suction [3].

According to the points raised, in geotechnical engineering cases under THM coupled condition, such as nuclear waste, geothermal structures, oil drilling and ect, thermo hydro mechanical properties is taken into account [17,18].

In 2019, Yan and Zhang [19] used the MSSA model to express the hydro mechanical behavior of these soils, extending it to a non-isothermal state, and simulating clay in isotropic and tri-axial conditions. In this model, conventional models that divide the soil into two parts, elastic and plastic, are used. This assumption does not show the soil behavior realistically, and to express the soil behavior more accurately, we need to use unconventional models that can be smooth transferred from the elastic to plastic state. Also in this model, the effects of hydraulic hysteresis are not considered. In 2021, Liu et al.,[15] presented a model of thermo-hydro mechanical behavior for unsaturated soils, which hydro mechanical behavior is developed based on the model of Wheeler et al., [20] but Wheeler's model had shortcomings in expressing hysteresis behavior and did not examine the effects of hysteresis carefully enough and limited to expressing simple hysteresis. Therefore more comprehensive models for considering hydraulic behavior and developing to thermo hydraulic is needed .

In the present study, a thermo-hydro mechanical model is presented to predict the behavior of unsaturated clay soils at different temperatures. This model based on the model presented by Ghasemzadeh et al. [1, 2]. the model is thermally developed and can investigate the interactions of temperature on stress-strain behavior as well as hydraulic behavior. The sub loading surface plasticity and boundary surface plasticity are developed with introduction of heat concept in the model to express the thermo-mechanical and thermo-hydraulic behavior in unsaturated soils, respectively. Also due to the importance of stress history on volume change [13, 21-23], the effects of OCR have been considered.

2. hydraulic behavior with boundary surface plasticity

By developing the theory of plasticity, Moraleetharan et al. [24] have presented an elasto-plastic model to express the behavior of unsaturated soils. In this model, he has considered normal effective stress P^* and matrix suction s

as stress variables as well as volumetric strain ε_v and water volume fraction n_w as strain variables. The initial values of suction in the wetting and drying path are expressed by the primary wetting and secondary drying paths, respectively. As shown in the SWCC curve in Fig. 1, all wetting and drying curves of subsequent cycles are limited to these two curves.

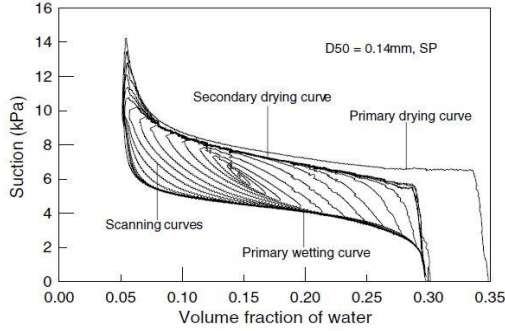


Fig. 1. measured SWCC by moraleetharan et al. [24]

Therefore, Moraleetharan by using the relationships presented by Feng and Ferdland [25], considered the empirical relationship for these two curves. Initial wetting curve:

$$s_{D0} = b_1 \left[\frac{n_{wsat} - n_w}{n_w - n_{wres}} \right]^{1/d_1} \quad (1)$$

Secondary drying curve:

$$s_{I0} = b_2 \left[\frac{n_{wsat} - n_w}{n_w - n_{wres}} \right]^{1/d_2} \quad (2)$$

In the above relations, n_{wsat} is the volumetric water content in zero suction and n_{wres} is the volumetric water content in the highest suction. b_1, d_1, b_2, d_2 are also four soil-related variables that fit the primary wetting and secondary drying curves. Since suction is a stress variable and is scalar, expressing the hysteresis behavior of the SWCC curve is a one-dimensional problem. At the beginning of each drying-wetting cycle, we only see the elastic capillary behavior, while during the other parts of the drying-wetting cycle, we see the elastoplastic capillary behavior. Moraleetharan et al., using the boundary surface plasticity, to express the behavior of elastic and plastic capillaries, the elastic Γ^e and plastic Γ^p capillary modulus have been expressed as follows:

$$\Gamma^e = \frac{ds}{dn_w^e} \quad (3)$$

$$ds = \Gamma^p dn_w^p \quad (4)$$

$$\Gamma^p = \Gamma^p(\delta, \delta_{in}) \quad (5)$$

$$\Gamma^p = \Gamma_0^p \left(1 + \frac{h \cdot \delta}{\langle \delta_{in} - g \delta \rangle} \right) \quad (6)$$

h and g are two soil parameters. According to Fig. 2. δ is equal to the vertical distance between the existing stress point and the corresponding wetting-drying curve and δ_{in} is equal to δ at the beginning of each wetting or drying path. Γ_0^p is equal to the amount of Γ^p when $\delta = 0$ and $\delta = \delta_{in}$ when $\Gamma^p = \infty$, and this means that elastic behavior at the beginning of the loading-unloading cycle is dominant ($\langle \rangle$ is Macaulay bracket).

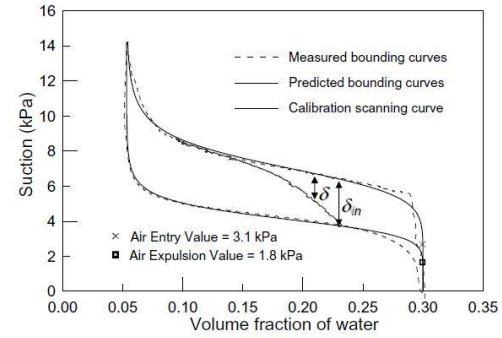


Fig. 2. SWCC by moraleetharan et al. [24]

3. Thermo-hydraulic development of boundary surface plasticity

In 2014, Zhou et al. [3] studied volumetric behavior changes of unsaturated soils under non-isothermal conditions. They presented an equation to consider changes in the degree of saturation in deformed soils under non-isothermal conditions. Accordingly, the effective degree of saturation of a soil under constant suction can be obtained based on the normalized cumulative distribution function of soil cavities $F(r)$, which varies between zero and one. $F(r)$ is actually dividing of volume of cavities by a radius greater than the desired radius r on the total volume of cavities. $F(r)$ Assuming that the network of soil pores has a simple geometric shape, can be obtained from the integral of the normalized pore size distribution

function $\bar{f}(r)$. It should be noted that $\bar{f}(r)$ does not include micro-cavities whose water remains after drying. If the cavities larger than the desired cavity radius dry out due matrix suction, the effective saturation degree is calculated as follows:

$$S_e = 1 - F(r) = 1 - \int_r^{r_{\max}} \bar{f}(r) dr \quad (7)$$

The key relationship between the normalized cumulative distribution function of soil cavities and soil water behavior is the same as the capillary law. Which is expressed as follows:

$$s = \frac{2\sigma_0^{\lg} \cos\theta}{r} = \frac{2\sigma^{\lg}}{r} \quad (8)$$

s is the suction of a matrix equal to the capillary pressure. $\sigma^{\lg}$ is the surface tension between gas and liquid, and θ is the contact angle between solid and liquid, which is usually considered to be zero for simplicity. It can also change with temperature and drying- wetting paths. The combination of Equations (7) and (8) shows the relationship between suction and saturation (SWCC equation). If we consider Van Genuchten equation to describe soil water behavior under normal laboratory conditions, then:

$$S_e = \frac{S_r - S_r^{\text{res}}}{1 - S_r^{\text{res}}} = (1 + (S/a)^n)^{-m} \quad (9)$$

Where S_r^{res} is residual degree of saturation and a , m , n are soil parameters related to the pore size distribution. The function of the normalized cumulative distribution of the soil cavities $F_0(r)$ according to the Van Genuchten equation at temperature T_0 from the combination of equations (7) to (9) is obtained as follows:

$$F_0(r) = 1 - \left(1 + \left(\frac{2\sigma_0^{\lg}}{a} \frac{1}{r}\right)^n\right)^{-m} \quad (10)$$

Where $\sigma_0^{\lg}$ is the interfacial tension at temperature T_0 . The soil is considered to have no deformation due to temperature changes, which means that the soil cavities do not change at different temperatures. $F_0(r) = F(r)$ where $F(r)$ is the cumulative function normalized to the desired temperature. Temperature changes in the capillary law affect only the interfacial tension, i.e. $r = \frac{2\sigma_T^{\lg}}{s}$ at T . The effective degree of saturation at temperature T is written as follows:

$$S_e = 1 - F(r) = \left(1 + \left(\frac{2\sigma_0^{\lg}}{a} \frac{s}{2\sigma_T^{\lg}}\right)^n\right)^{-m} \quad (11)$$

where $\sigma_T^{\lg}$ is the interfacial tension at temperature T . Philip and De Vries [5] presented a linear function to investigate the effects of temperature on interfacial tension.

$$\sigma_T^{\lg} = a' + b'T \quad (12)$$

where a' and b' are its parameters. Grant and Salehzadeh [26] calibrated its parameters for levels between pure water and air which are as follows:

$$a' = 0.1177(\text{Nm}^{-1}), b' = -0.0001535 (\text{Nm}^{-1}\text{K}^{-1})$$

After replacing equation (12) in equation (11):

$$S_e = 1 - F(r) = \left(1 + \left(\frac{\beta' + T_0 s}{\beta' + T a}\right)^n\right)^{-m} \quad (13)$$

which $\beta' = a'/b' = -766.7\text{K}$ was first considered by Grant and Salehzadeh to model the effects of temperature on soil water behavior. To investigate the effects of temperature on the primary wetting curve and secondary drying curve, the method proposed by Zhou et al. [3] were used to obtained a relation in terms of n_w , s and T that correlate with the curves presented by the Moraleetharan in the initial temperature conditions. We use the effective degree of saturation equation.

$$S_e = \frac{S_r - S_r^{\text{res}}}{1 - S_r^{\text{res}}} \quad (14)$$

$$n_w = \frac{S_r e}{1 + e} \rightarrow S_r = \frac{1 + e}{e} n_w \quad (15)$$

By replacing equation (15) in equation (14):

$$S_e = \frac{\frac{1+e}{e} n_w - \frac{1+e}{e} n_w^{\text{res}}}{\frac{1+e}{e} n_w^{\text{sat}} - \frac{1+e}{e} n_w^{\text{res}}} = \left(\frac{n_w - n_w^{\text{res}}}{n_w^{\text{sat}} - n_w^{\text{res}}}\right) \quad (16)$$

inwhich for saturated situation we used following relation based on equation (15):

$$\frac{1+e}{e} n_w^{\text{sat}} = 1 \quad (17)$$

To calculate the matrix suction in Equation (13), we place the suction on one side,

$$s = a * ((S_e)^{-1/m} - 1)^{1/n} * \frac{\beta' + T}{\beta' + T_0} \quad (18)$$

inwhich

$$(S_e)^{-1/m} = \left(\frac{n_w - n_w^{\text{res}}}{n_w^{\text{sat}} - n_w^{\text{res}}}\right)^{-1/m} \quad (19)$$

By replacing equation (19) in equation (18):

$$s = a \frac{\beta' + T}{\beta' + T_0} \left(\left(\frac{n_w - n_w^{\text{res}}}{n_w^{\text{sat}} - n_w^{\text{res}}} \right)^{-1/m} - 1 \right)^{1/n} \quad (20)$$

According to Equation (15) :

Which by replacing in Equation (20) :

$$s = a \frac{\beta' + T}{\beta' + T_0} \left(\left(\frac{n_w - n_w^{\text{res}}}{n_w^{\text{sat}} - n_w^{\text{res}}} \right)^{-1/m} - 1 \right)^{1/n} \quad (21)$$

3.1 Thermo-hydraulic behavior of the proposed model

Equation (21) can be applied to the boundary curves of primary wetting and secondary drying. The above equation is derived from the fundamental equations and the widely used Ven Genuchten equation, which, despite the addition of two other parameters and can predict the hydraulic behavior of boundary curves and hysteresis curves with respect to temperature changes.

Initial wetting curve:

$$s = b_1 \frac{\beta' + T}{\beta' + T_0} \left(\left(\frac{n_w - n_w^{\text{res}}}{n_w^{\text{sat}} - n_w^{\text{res}}} \right)^{a_1} - 1 \right)^{1/d_1} \quad (22)$$

Secondary drying curve:

$$s = b_2 \frac{\beta' + T}{\beta' + T_0} \left(\left(\frac{n_w - n_w^{\text{res}}}{n_w^{\text{sat}} - n_w^{\text{res}}} \right)^{a_2} - 1 \right)^{1/d_2} \quad (23)$$

The values $n_{w_{\text{sat}}}$, $n_{w_{\text{res}}}$ are obtained using the minimum and maximum values n_w in the primary wetting and secondary drying curves, respectively. Values $a_1, b_1, d_1, a_2, b_2, d_2$ are also obtained by fitting the wetting and drying curves to the primary wetting and secondary drying curves.

3.2 Thermo-mechanical behavior with sub-loading surface plasticity

There are various unconventional elastic-plastic model to study the behavior of soils, among which we can refer to the sub-loading surface model proposed by Hashigushi et al. [27, 28]. In the sub-loading surface model, because inside of yield surface does not have a fully elastic behavior, the yield renamed to normal yield surface. The sub-loading surface is also introduced as the surface that always passes the existing stress and has a shape similar to the normal yield surface and the same directions as the main stress axis directions. Fig. 3. shows the normal yield surface and subloading yield surface.

The center of similarity of these two surfaces in

this model is the center of the stress space, i.e. the zero stress point. The similarity of these two surfaces gives the following results:

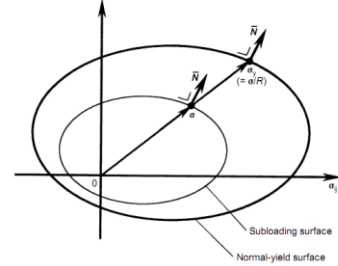


Fig. 3. the normal yield surface and sub-loading yield surface [28]

1. All lines that connect a stress point from the sub-loading surface to the corresponding point on the normal yield surface, pass through the center of similarity.
2. The ratio of the length of all lines connecting two points on the sub-loading surface to the length of the lines connecting the two points on the normal yield surface is a constant value at any given moment, called the similarity ratio. This ratio also indicates the size ratio of the two surfaces (R).

This ratio is always between zero and one, where $R = 0$ indicates the zero size of the sub-loading surface and $R = 1$ indicates the conformity of the sub-loading yield surface and normal yield surface. This ratio always increases when the plastic strain occurs so that the sub-loading surface coincide the normal yield surface, after that ratio remains constant to one, and sub-loading surface and the normal yield surface resize in the same way. This similarity ratio in the following loading model indicates the pre-consolidation ratio of the soil. The soil is normally consolidated when $R = 1$ and over consolidated soil when $R < 1$. In the proposed thermo-hydro-mechanical model in the triaxial stress space, we have used sub-loading surface to express the thermo-mechanical behavior of unsaturated soil. As stated earlier, in the following model, the yield function of sub-loading surface is expressed by Equ. (24) :

$$f(\sigma^*) = P^*(1 + X^2) = RF \quad (24)$$

in which

$$X = \frac{\|\eta\|}{m} \quad (25)$$

$$\|\eta\| = \frac{\|\sigma'\|}{P^*} \quad (26)$$

$$\|\sigma'\| = \sqrt{\frac{2}{3}}q \quad (27)$$

In the above relations, m is equal to the slope of the critical state line on the plane $p^* - q$ and σ' is considered as a tensor of the deviation stress components. According to the above relations, the yield function in this plane can be expressed as follows:

$$f(p^*, q) = p^* + \frac{2q^2}{3p^*m^2} = RF \quad (28)$$

in the above relation F is equal to the hardening function and is expressed by the following relation:

$$F = F_0 \exp\left(\frac{H}{\lambda_s - K}\right) \quad (29)$$

F' is also equal to:

$$F' = \frac{F}{\lambda_s - K} \quad (30)$$

In Equation (29) H, F_0 are equal to the initial value of F and the soil hardening parameter, respectively. λ_s and K are equal to the slope of the normal consolidation curve and the swelling curve on a plane $\ln V - \ln p^*$ respectively, where V is equal to the volume of the soil. Based on the experimental results, the slope of the critical line is a function of temperature, for which the following equation is considered.

$$M_T = M \exp[\beta(T - T_0)] \quad (31)$$

To study these changes, Cekerevac and Laloui[29] conducted experiments on Boom clay that showed an increase in the slope of the critical line with temperature. By placing $\beta = 0.0011$ the correctness of M behavior is observed.

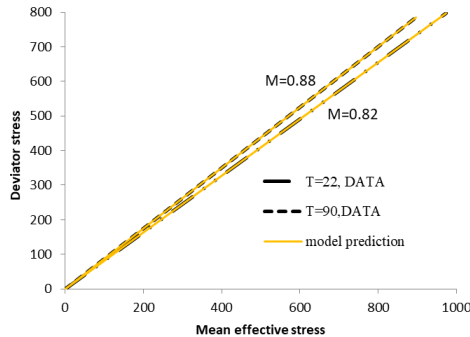


Fig. 4. Comparison of critical line slope at 22 and 90 ° C [29]

As stated before, the yield surface in the stress and temperature plane shows thermal softening, which is defined as follows [16].

$$P_{0T} = P_0 \exp(-\beta \Delta T) \quad (32)$$

where β is the parameter describing the variations of the yield surface with temperature. Considering the effect of cohesion on the increase of shear and tension strength in soil and according to:

$$P_s = Ls \quad (33)$$

where parameter L is describing the increase in cohesion with suction. Equation (28) is rewritten as follows:

$$f(p^*, q) = p^* + \frac{2q^2}{3(p^* + p_s)m^2} = RF \quad (34)$$

3.2.1 The coupled hardening rules

Due to the coupled thermo-hydro-mechanical relationships described in the previous section, the hardening in unsaturated soils changes as a function of the plastic volumetric strains and the plastic volumetric water content.

$$\dot{H} = d\varepsilon_v^p - dn_w^p \quad (35)$$

Also, due to the coupled thermo-hydro-mechanical relationships and since the values of the primary suction increase and decrease curves are always on the primary wetting and secondary drying curves, the hardening of the suction increase and suction decrease curves can be derived by deriving the relationship (22) and (23).

$$dS_{D0} = \frac{\partial S_{D0}}{\partial n_w^p} dn_w^p + \frac{\partial S_{D0}}{\partial \varepsilon_v^p} d\varepsilon_v^p \quad (36)$$

$$dS_{I0} = \frac{\partial S_{I0}}{\partial n_w^p} dn_w^p + \frac{\partial S_{I0}}{\partial \varepsilon_v^p} d\varepsilon_v^p \quad (37)$$

3.2.2 Elasto-plastic relations

In this section, the strain formulations (volumetric and shear) and the elastic and plastic volumetric water content used in this model are presented.

Elastic Volumetric strain, shear strain and volumetric water content

Since the temperature changes can also affect the soil volume change independent of the stress level, then:

$$d\varepsilon_V^e = \frac{kdp^*}{Vp^*} + \kappa_T dT \quad (38)$$

$$d\varepsilon_q^e = \frac{dq}{2G} \quad (39)$$

$$dn_w^e = \frac{ds}{\Gamma^e} \quad (40)$$

In the above relations, k is equal to the slope of the elastic swelling line in the plane $V - \ln p^*$, Γ^e modulus of elastic capillary and G is the shear modulus, which is obtained from the following equation.

$$G = \frac{3P^*(1-2\nu)}{2K(1+\nu)} \quad (41)$$

In the above relation ν is Poisson's coefficient.

Plastic Volumetric and shear strain

According to the relations provided by Hashigushi, we have:

$$tr\left(\frac{\partial f(\sigma^*)}{\partial \sigma^*} \cdot \dot{\sigma}^*\right) = U \cdot \lambda \cdot F + RF' \dot{H} \quad (42)$$

$$\frac{R \cdot F \cdot \bar{N}^T \cdot d\sigma^*}{\bar{N}^T \cdot \sigma^*} = U \cdot \lambda \cdot F + R \cdot F' (\lambda h - dn_w^p) \quad (43)$$

We have also considered the value of plastic volumetric strains as follows:

$$d\varepsilon_V^p = \lambda h \quad (44)$$

$$h = \frac{\partial f(p^*, q)}{\partial p^*} \left/ \left\| \frac{\partial f(\sigma^*)}{\partial \sigma^*} \right\| \right. \quad (45)$$

According to Equation (43) the value is obtained as follows:

$$\lambda = \frac{\bar{N}^T \cdot d\sigma^*}{M^P} + \frac{dn_w^p}{L^P} \quad (46)$$

$$M^P = \left(\frac{U}{R} + \frac{F'}{F} h \right) \bar{N}^T \cdot \sigma^* \quad (47)$$

$$L^P = \frac{U \cdot F}{R \cdot F'} + h \quad (48)$$

$$\sigma^* = \begin{pmatrix} p^* \\ q \end{pmatrix}, d\sigma^* = \begin{pmatrix} dp^* \\ dq \end{pmatrix} \quad (49)$$

$$d\varepsilon^p = \begin{pmatrix} d\varepsilon_V^p \\ d\varepsilon_q^p \end{pmatrix} \quad (50)$$

$$\bar{N} = \begin{pmatrix} \frac{\partial f(p^*, q)}{\partial p^*} \left/ \left\| \frac{\partial f(\sigma^*)}{\partial \sigma^*} \right\| \right. \\ \frac{\partial f(p^*, q)}{\partial q} \left/ \left\| \frac{\partial f(\sigma^*)}{\partial \sigma^*} \right\| \right. \end{pmatrix} \quad (51)$$

$$\frac{\partial f(p^*, q)}{\partial p^*} = 1 - \frac{2q^2}{3m^2(p^* + p_s)^2} \quad (52)$$

$$\frac{\partial f(p^*, q)}{\partial q} = \frac{4q}{3m^2(p^* + p_s)} \quad (53)$$

Which according to associated flow rule:

$$d\varepsilon^p = \lambda \cdot \bar{N} \quad (54)$$

Using Equation (54), the volumetric and shear components of plastic are obtained with the following equations:

$$d\varepsilon_V^p = \frac{A}{M^P} dp^* + \frac{B}{M^P} dq + \frac{D \cdot dn_w^p}{L^P} \quad (55)$$

$$d\varepsilon_q^p = \frac{B}{M^P} dp^* + \frac{C}{M^P} dq + \frac{E \cdot dn_w^p}{L^P} \quad (56)$$

Plastic volumetric water content

Derived from the equation $n_w = V_w/V$, the rate of change of the volumetric water content is obtained as follows:

$$dn_w = \frac{dV_w}{V} + n_w \cdot d\varepsilon_{Vm} \quad (57)$$

In the above equation, V_w is equal to the volume

of water and V is equal to the total volume of soil and ε_{Vm} is equal to the volumetric strains resulting from mechanical loads. The above relation states that any change in the volumetric water content is due to a change in the volume of water between the soil cavities and a change in the volume of the soil skeleton. It is noteworthy that according to the concept of derivation, the first part of Equation (57) considers only the change in water volume between cavities due to suction and temperature, and the change in soil skeleton volume is assumed to be constant (hydraulic behavior). The second part considers only the change in the volume of the soil skeleton and the change in the volume of water between the cavities is assumed to be fixed (mechanical behavior). Similarly, it can be said that the rate of change in the plastic volumetric water content is obtained from the plastic volume changes of the soil cavities.

$$dn_w^p = \frac{dV_w^p}{V} + n_w \cdot d\varepsilon_{Vm}^p \quad (58)$$

According to the above point, the plastic volume change of water between cavities is only a function of the change in suction and the change in plastic volume of soil cavities is only a function

of the mechanical part of the plastic volumetric strain. Water expands during heating, which can be defined by thermal volume expansion of water coefficient, since the temperature volume change of water is as follows:

$$dV_w = V_w \alpha_w dT \quad (59)$$

$$\frac{dV_w^P}{V} = \frac{ds}{\Gamma^P} + \alpha_w dT \quad (60)$$

$$d\varepsilon_{vm}^P = \frac{A}{M^P} dp^* + \frac{B}{M^P} dq \quad (61)$$

$$dn_w^P = \frac{ds}{\Gamma^P} + \frac{A.n_w}{M^P} dp^* + \frac{B.n_w}{M^P} dq + \alpha_w dT \quad (62)$$

which equation (60) has been presented by Moraleetharan et al., and Γ^P is equal to the plastic modulus of capillary and can be calculated from Equ. 6. In which Γ_0^P is also obtained from the following equations:

$$\Gamma_0^P = \frac{ds_{D0}}{dn_w} \quad (63)$$

$$\Gamma_0^P = \frac{ds_{I0}}{dn_w} \quad (64)$$

According to Xiong et al. [30] which studies the thermo-mechanical behavior of soils, the concept of equivalent stress can be used to express the thermo-mechanical behavior. In many experiments under constant mean stress conditions σ_m , the temperature rises from its initial value T_0 to T , causing thermo-elastic volumetric strain changes (expansion), such as when materials are subjected to actual unloading mean stresses $\Delta\sigma_m$. Therefore, it is assumed that the thermo-elastic volumetric strain is created by increasing the hypothetical stress $\Delta\tilde{\sigma}_m$, which is called the equivalent stress increase. Considering the concept of stress equivalent to the relationship between $\Delta\tilde{\sigma}_m$ and T is obtained as follows:

$$\Delta\tilde{\sigma}_m = \sigma_m \exp\left[\frac{3\alpha_T(T-T_0)(1+e_0)}{\kappa}\right] - \sigma_m \quad (65)$$

which κ is the swelling index and e_0 porosity in the initial state. As a result, the equivalent stress is expressed as follows:

$$\tilde{\sigma}_m = \sigma_m + \Delta\tilde{\sigma}_m = \sigma_m \exp\left[\frac{3\alpha_T(T-T_0)(1+e_0)}{\kappa}\right] \quad (66)$$

The relationship between temperature and thermo-elastic strain is shown in Figure 5.

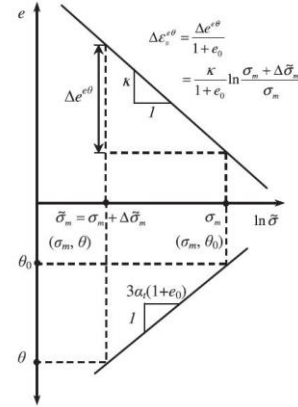


Fig. 5. Relationship between temperature and thermal elastic strain [30]

As a result of the above statement, we define the concept of equivalent sub-loading surface with respect to temperature. According to the above cases, effective stress is defined as follows:

$$p^* = (p + n_w.S) \exp[-\beta(T - T_0)] \quad (67)$$

4. Experimental validation

In this section, numerical simulation are compared with the experimental results of Uchaipicah and Khalili [4] on compacted soil samples of Bourke region of Australia. Model validation includes:

- 1-Temperature controlled wetting/drying tests,
- 2-Temperature/Suction controlled isotropic tests,
- 3-Temperature controlled soil water characteristic curves, and
- 4-Suction controlled heating/cooling tests.

The soil was compacted to a dry weight of 15.3 kN/m³ and moisture content of 10.5%. The tests were performed using a modified Bishop–Wesley tri-axial cell capable of independent measurement and control of pore air pressure, pore water pressure, and temperature at the sample boundaries. Each sample was loaded to an isotropic effective stress of 200 kPa to remove the effects of sample disturbance due to the setting-up process, and then unloaded to 50 kPa. All samples tested had an initial pre-consolidation pressure of 200 kPa and an over consolidation ratio of 4. Matrix suction values ranged from zero to 300 kPa and temperature values ranged from 25 to 60 °C. In the process of increasing the cell water temperature, the net cell pressure was 50 kPa. All parameters used in the mode are given in Table 1.

Table 1. Parameters used in presented model

Mechanic and Hydraulic Parameters			
Parameter	Value	Parameter	Value
λ_s	0.09	F_0	200 kPa
ν	0.3	u	60
L	0.1	m	1.17
Γ^e	-1150 kPa	k	0.006
h	10	b_1	22
g	0.1	b_2	105
$n_{w_{sat}}$	0.611	d_1	1.635
$n_{w_{res}}$	0.305	d_2	4.8
a_1	1.0006	a_2	1.006
Temperature Parameters			
β'	-493.5	α_w	0.00018
K_T	-0.00004	η_l	0.000514
β	0.0002	γ	0.75
α	0.0075		

Fig. 6.(a) Comparing the results of the proposed model with the experimental results of Sun et al.[31] which this comparison was in the model of Ghasemzadeh and Ghoreishian [1] and updated by new thermo-hydraulic model that can predict hydraulic behavior in non-isothermal condition. Since this model based on the basis formulation of unsaturated soils, in addition to predicting the hydro-mechanical states, can be developed into the thermo-hydraulic states in soils. Fig. 6.(b) shows the effects of temperature on boundary curves. According to the proposed analytical model, it is observed that with increasing temperature, the volume of water decreases at a constant suction and the temperature changes are limited to boundary curves.

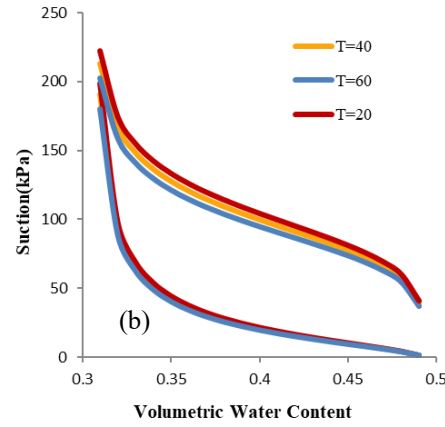
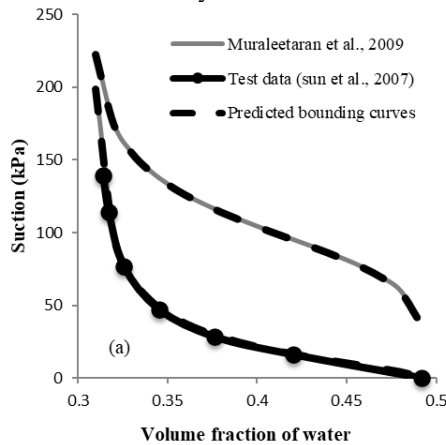


Fig. 6. Boundary curves: (a) comparing with experimental data, (b) temperature effect

The effective pre-consolidation pressure gradually increased with matric suction. The changes in yield stress (pre-consolidation) at different matric suctions and temperatures are shown in Fig. 7. It is clear that with increasing matric suction (drying) we will have an increasing trend. Also, as the temperature increases, the rate of these changes decreases.

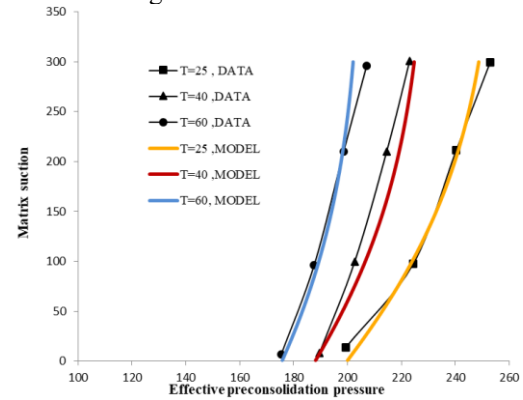


Fig. 7. Loading collapse curve at different temperatures

According to the results, the value of M is 1.17, by taking this value into account, yield surface diagrams are drawn. Changes in the yield surface with temperature and matric suction are shown in Fig. 8. At constant suction, an increase in temperature causes the yield surface to contract, which is fully consistent with laboratory evidence. This phenomenon is more noticeable in higher matric suctions. On the other hand, increasing the suction at a constant temperature causes the yield surface to expand, these effects are less noticeable

at higher temperatures.

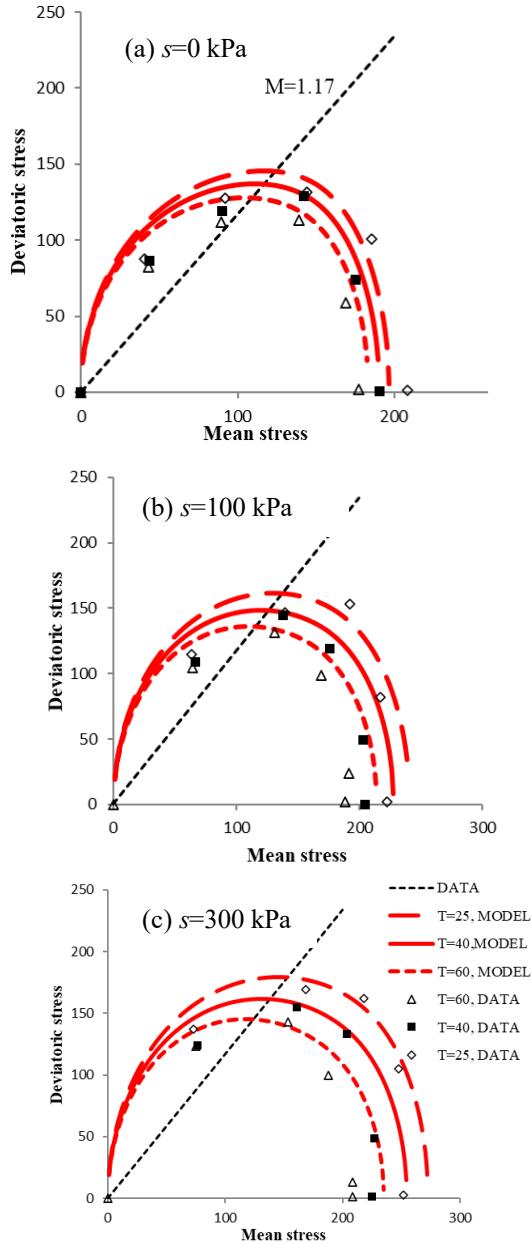


Fig. 8. Yield surface changes in different temperatures and suctions: (a) $s=0$ kPa, (b) $s=100$ kPa, (c) $s=300$ kPa

Fig. 9. and Fig. 10. show that the normal compression line shifts to lower stresses as the temperature increases. It also shows that the slope of unloading-line κ is independent of temperature and matric suction. The value of λ decreases with increasing matrix suction.

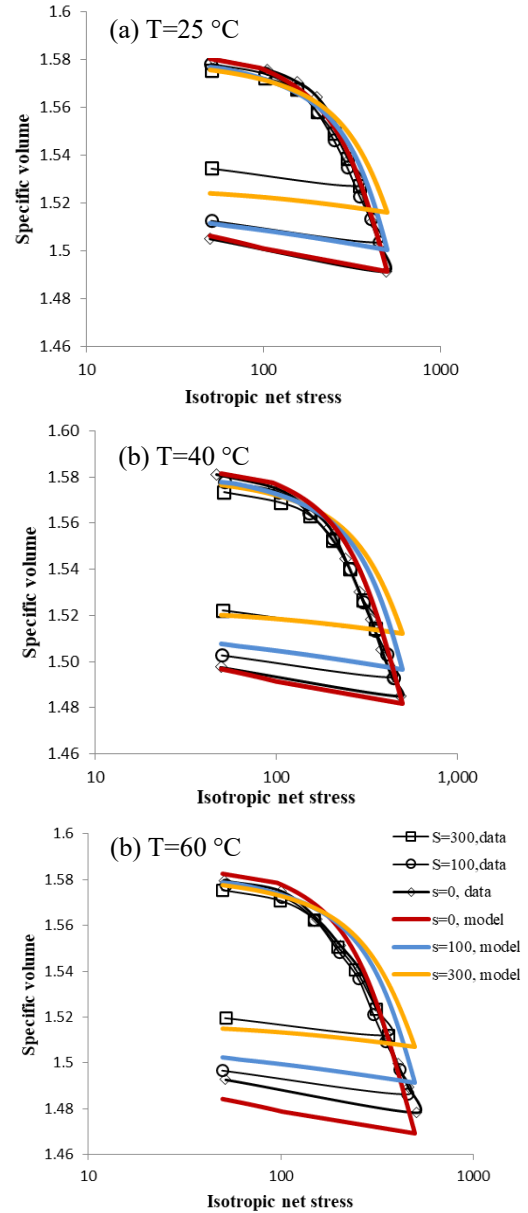


Fig. 9. Volume changes in different suctions for Temperatures: (a) $T=25$, (b) $T=40$, (c) $T=60$ °C

The relationship between the degree of saturation and the matric suction, known as the soil water characteristic curve, is shown in Fig.11. for different temperatures. Changes in saturation are determined by the volume of water removed from the soil and volume changes in sample. The soil water characteristic curve at constant suction and temperature is independent of the stress level, although the degree of saturation decreases with increasing temperature.

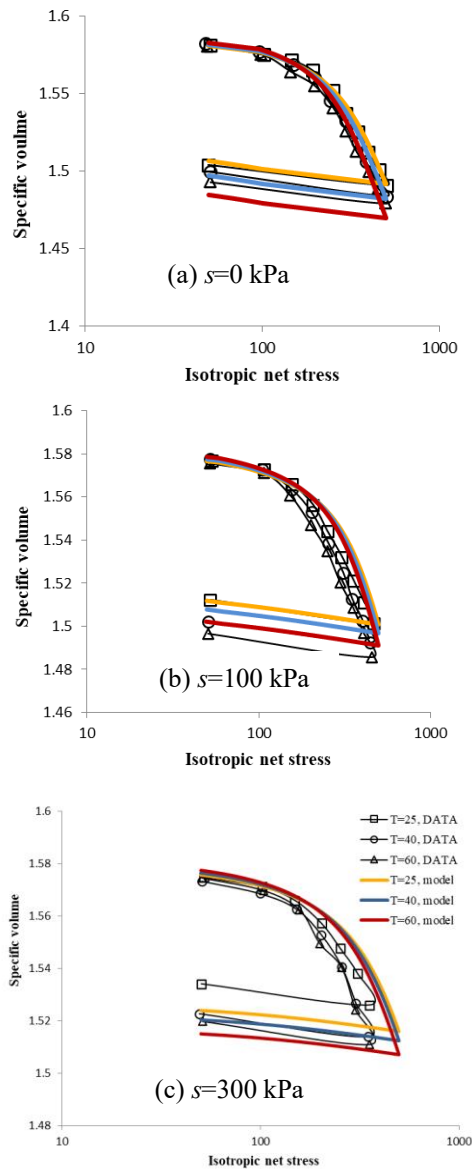


Fig.10. Volume changes in different temperatures for suctions: (a) S=0 kPa, (b) S=100 kPa, (c) S=300 kPa

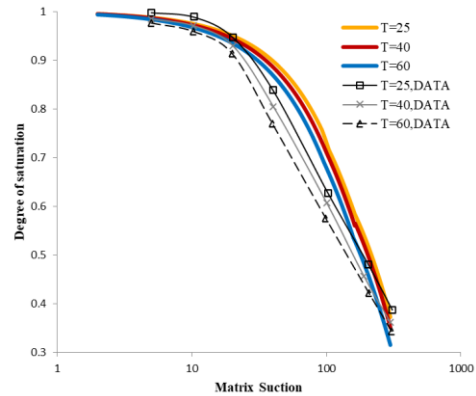


Fig.11. soil water characteristic curve change by temperature

The specific volume results against temperature changes are shown in Fig. 12. for different suctions. As can be seen, in saturated soil ($s=0$) due to temperature increase, expansion occurs at high over-consolidation ratio (OCR=4) and returns to its initial state almost by cooling. The tendency to dilate decreases with increasing net stress and is observed at lower over-consolidation ratios (OCR=1), due to heating contraction will occur, which is not reversible due to cooling. The lower the over consolidation ratio (higher net stress), the higher the contraction rate. The same trend exists for unsaturated soils and the amount of contractions increases with increasing suction. The results also show that the cooling path are parallel to each other, and this shows that the thermal expansion coefficient of the soil solid structure is independent of net suction and stress. In normal consolidated soils and slightly over-consolidated soils, thermal softening occurs which decreases with increasing over consolidation ratio. There is no thermal collapse in soils with high over-consolidation ratio and their volume changes due to heat are in the form of elastic expansion.

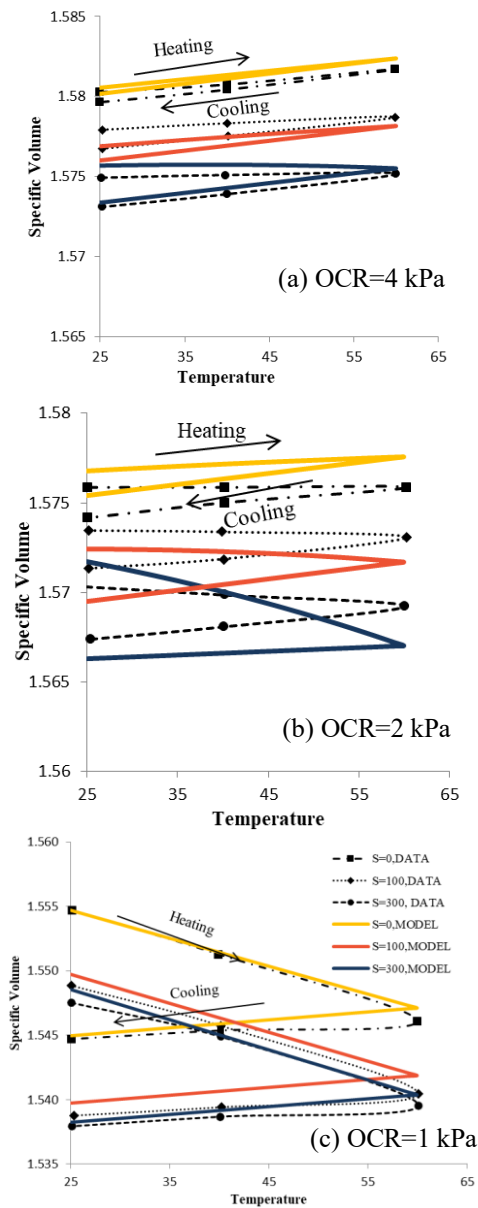


Fig.12. Specific volume change cause by temperature change in different suctions: (a) OCR=4, (b) OCR=2, (c) OCR=1

As the effective stress decreases, the soil shrinks at a higher temperature (Figure 13). As can be seen, the soil with a over-consolidation ratio of 2 at about 55 °C is transferred from expansion to shrinkage, while in soils with over-consolidation ratio of 4 this transfer occurs at a temperature of 85 °C. As can be seen in the Fig. 11 to 13, the difference between the experimental data and the model results is not significant, and the model results are within an acceptable range.

It should be noted that this article focuses on presenting the results of thermal modeling; therefore, stress-strain curves and cyclic loading results are not included here. For further details, the reader may refer to the relevant references [1, 2].

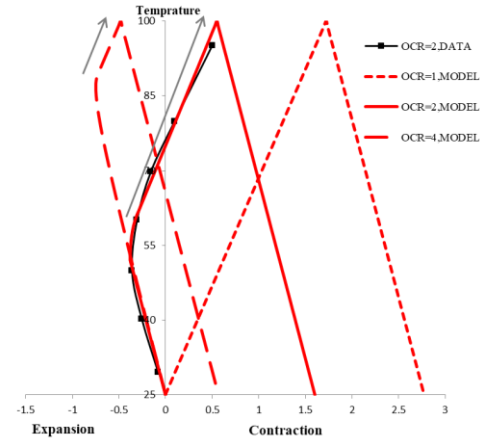


Fig. 13. Thermal volumetric strain change by temperature change in different OCRs[32]

5. Conclusion

In many geotechnical engineering works, soil is exposed to high temperatures, and as the temperature increases, a series of microscopic and microscopic changes occur in the soil. Therefore soil behavior prediction model, especially fine-grained soils at high temperatures is needed. In this research, a coupled elastoplastic behavioral model is presented to express the behavior of unsaturated soils under tri-axial loading conditions. The presented model is an extension of the behavioral model proposed by Ghasemzadeh et al.[2] that can express the thermal behavior under cyclic mechanical loading and drying-wetting cycles. In the proposed model, the boundary surface plasticity was used to express the hydraulic behavior and by using fundamental thermal equations it developed to thermo-hydraulic. To study the thermomechanical behavior by assuming temperature as internal stress in sub-loading surface plasticity, the thermal effect were simulated.

The R-parameter indicates the over-consolidation ratio in sub-loading surface plasticity. As observed, at high over-consolidation ratios expansion was observed which was extremely reversible by cooling, but at lower ratios, irreversible contractions were observed due to heating. The slope of the cooling paths in the soils

was the same, indicating that the thermal expansion coefficient of the solid structure of the soil was not dependent on suction and stress. Contractions increase at high suctions. Over-consolidated soils after expansion at high temperature transferred to the shrinkage state, in which at the higher suction and the lower over-consolidation ratio, occurs at lower temperature. Soils can yield under different thermal, hydraulic and mechanical loads. Changes in each of these three cases affect the soil yield position. As observed, the yield level increased with increasing soil suction and decreased with increasing temperature. With increasing temperature, the soil water content capacity decreased and in constant suction, the saturation degree decreased due to heating, which is attributed to the soil volume change. Due to the calibration performed and verification to the laboratory data, the model can be responsible for over-consolidated and normal consolidated soils.

6. References

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