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Extended Abstract

Development of Mohr-Coulomb criterion elastoplastic integration algorithm scheme for rock

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Abstract

Elastoplastic criteria are very important in many topics related to petroleum geomechanics, geotechnics, and rock mechanics. Due to the importance of these criteria, their numerical implementation is considered essential. Although some of the existing software includes the stated criteria, due to the lack of access to the coding core of the software, the accuracy of the modeling done with them is practically not fully assured. Therefore, considering the importance of these criteria and of course their complexity for implementation, in this research a comprehensive numerical model to improve the elastoplastic integration algorithm of the Mohr-Coulomb criterion was presented and described in detail. The proposed integration algorithm includes two steps elastic trial step and the plastic corrector step. In the proposed model, if the elastic trial step is in the elastic region or on the yield surface, the answer of elasticity is accepted. Otherwise, if the trial stress in the first step cannot confirm the acceptable conditions, it is provided by the return-mapping algorithm. This process is done for all surfaces of the Mohr-Coulomb criterion and the top of the model comprehensively and of course separately until the Mohr-Coulomb model can present the elastoplastic behavior of the material during loading. The presented model for rock was investigated and the validity of the proposed model was confirmed by comparing the numerical results with the experimental data.

1. Introduction

In petroleum geomechanics, geotechnics, and rock mechanics, elastoplastic constitutive models with multiple yield surfaces are widely used in engineering applications, among which the Mohr-Coulomb criterion is one of the most common. On the other hand, the complexities of implementing these models require the development of the most appropriate elastoplastic integration algorithm, which, in addition to convergence, has a high convergence speed. The mentioned cases show the complexity of this issue. Therefore, in this research, considering the importance of the subject, a comprehensive scheme for the elastoplastic integration algorithm of the Mohr-Coulomb criterion is proposed. The proposed approach for the Mohr-Coulomb criterion is confirmed and validated by comparing the numerical results of the Mohr-Coulomb model with the laboratory results.

2. Methodology

The Mohr-Coulomb model consists of three key principles, namely: (a) the elastic law, (b) the yield criterion., and (c) the flow rule. To use the Mohr-Coulomb model in geomechanical software or developed codes, this model must be implemented. The implementation of this model is done using the numerical integration algorithm which is divided into two main stages. These steps are the elastic trial step and the plastic corrector step (or return-mapping algorithm). If the elastic

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trial step is in the elastic region or on the yield surface, the answer is accepted. Otherwise, if the trial stress in the first step fails to confirm the acceptable condition of plasticity, it is predicted by the return-mapping algorithm [16] on the yield surface. Given the four possible descriptions of the Mohr-Coulomb flow rule, the return-mapping algorithm based on Euler's theory has four clearly distinct forms that depend on the location of the updated stress at the yield surface.

3. Results and Conclusions

The description provided for the Mohr-Coulomb criterion and the integration algorithm developed in this research provides this possibility for readers interested in implementing the Mohr-Coulomb model. The developed integration algorithm only consists of two the elastic trial step and the plastic corrector step or return-mapping algorithm. The advantage of this algorithm is its numerical stability, which will usually lead to fast convergence and a stable algorithm. The developed integration algorithm can be used as a suitable infrastructure to provide elastoplastic models in developed codes or software under development.

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